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Model Questions

B.Sc Semester VI

Paper – XIV

Short Answer type Questions (Vector Integration)
(Group A)

① Evaluate $\int_C F\cdot dr$, where $F = x^2\vec{i} + y^3\vec{j}$ and Curve C is the arc of the parabola $y = x^2$ in the x-y plane from (0,0) to (1,1).

② If $F = (2x+y)\vec{i} + (3y-x)\vec{j}$, evaluate $\int_C F\cdot dr$ where C is the curve in the xy- Plane consisting of the straight lines from (0,0) to (2,0) and then to (3,2).

③ Evaluate $\iint_S F\cdot n\, dS$, where $F = yz\,\vec{i} + zx\,\vec{j} + xy\,\vec{k}$ and S is that part of the surface of the sphere $x^2+y^2+z^2=1$ which lies in the first octant.

④ state and prove Green's theorem.

⑤ Evaluate $\oint_C F\cdot dr$ by stoke's theorem where $F = y^2\vec{i} + x^2\vec{j} - (x+z)\vec{k}$ ~~where~~ and C is the boundary of the triangle with vertices at (0,0,0), (1,0,0), (1,1,0).

Short Questions From Linear Algebra
(Group B).

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Group B [Linear Algebra]

Reference Book - J.N Sharma & A.R Vasishtha.

⑥ Define Vector Subspaces and prove that the intersection of any two Subspaces W_1 and W_2 of a vector space $V(F)$ is also a Subspace.

⑦ Define vector Space and let $V(F)$ be a vector space and 0 be the zero vector of V, Then

(i) $a0 = 0 \quad \forall \ a \in F$
(ii) $a(-\alpha) = -(a\alpha) \quad \forall \ a \in F \quad \forall \ \alpha \in V$
(iii) $(-a)\alpha = -(a\alpha) \quad \forall \ a \in F$.

⑧ Define (a) Basis of a vector Space.
(b) Linear Dependence and Linear Independence of Vectors.

⑨ Show that in the vector space $V_n(F)$, the System of n vectors

$$e_1 = (1, 0, 0 \cdots 0)$$

$$e_2 = (0, 1, 0 \cdots 0)$$

$$\cdots \cdots$$

$$e_n = (0, 0, \cdots 0, 1)$$ is a basis of $V_n(F)$.

⑩ Define Inner product Space and give the example of inner product Space.

Long Answer type Questions

Group - A (Vector Integration)

① Verify Green's theorem in the plane for $\oint_C (xy + y^2)\, dx + x^2 dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

② Verify divergence theorem for

$$F = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$$ taken

over the rectangular parallelopiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$.

③ State and Prove Stoke's theorem.

④ Verify Stoke's theorem for
$$F = (x^2 + y^2)\vec{i} - 2xy\vec{j}$$

taken round the rectangle bounded by
$$x = \pm a, \quad y = 0, \quad y = b.$$

⑤ Evaluate $\oint_C F \cdot dr$ by Stoke's theorem where
$F = y^2 \vec{i} + x^2 \vec{j} - (x+z)\vec{k}$ and C is the boundary of the triangle with vertices at $(0,0,0)$, $(1,0,0)$, $(1,1,0)$.

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Long Questions of Vector Space.
(Group - B)

(6) Prove that the union of two subspaces is a subspace if and only if one is contained in the other.

(2) Show that the system of three vectors $(1,3,2)$ $(1,-7,-8)$, $(2,1,-1)$ of $V_3(R)$ is linearly dependent.

(3) If W_1, W_2 are two subspaces of a finite dimensional vector space $V(F)$, then $\dim(W_1+W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$. prove it.

(4) Let U and V be vector spaces over the field F and let T be a linear transformation from U into V. Suppose that U is finite dimensional, Then prove that
$$\text{rank } T + \text{nullity } T = \dim U.$$

(5) Apply the Gram - Schmidt process to the vectors
$$B_1 = (1,0,1),\ B_2 = (1,0,-1)$$
$$B_3 = (0,3,4)$$ to obtain an orthonormal basis for $V_3(R)$ with the standard inner product.

[Reference Book -
Vector Calculus - J.N Sharma & A.R Vasistha
Linear Algebra - J.N Sharma & A.R Vasistha.